Session Versus Baseline GPS Processing

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BIOGRAPHY

Michael R. Craymer joined the Geodetic Survey of Canada in late 1991 as a Senior Geodetic Engineer. Prior to this he was both a member of the research staff in Geodetic Research Laboratory at the University of New Brunswick as well as General Manager and Vice-President of Geodetic Research Services, a small consulting and software development company. He received his Master's degree in geodesy from the University of Toronto in 1984 and is presently completing his Ph.D. dissertation in his spare time.

Norman Beck has been with the Geodetic Survey of Canada since 1984, where he has worked on GPS precise positioning related systems development, data processing and analysis and is now chief of the Positional Control Section. For five years prior, he worked at Nortech Surveys developing early generation GPS positioning and integrated navigation systems. He is a co-author of "Guide to GPS Positioning" and an Ontario Land Surveyor.

ABSTRACT

Many software packages used for static GPS surveying are based only on baseline processing, where the mathematical correlations between simultaneously observed baselines are neglected. Moreover, all possible baselines, including so-called “trivial” baselines that are linear combinations of others, are often combined together in a 3D adjustment. To date there have been no known conclusive tests that permit an objective evaluation of the effect of ignoring these correlations and including trivial baselines. Realizing that many GPS users will have access to only baseline processing software, we investigate processing strategies that could be used to provide results equivalent to session processing. In principle, session solutions which account for all mathematical correlations are preferable since they also allow for orbit improvement, spatial modeling of atmospheric effects and enable easier ambiguity resolution. On the other hand, baseline processing with all trivial baselines significantly distorts the formal accuracies by artificially increasing the redundancy in the model, resulting in overly optimistic covariance matrices. It is shown that using all possible baseline solutions (with the covariance matrix scaled by n/2, where n is the number of simultaneously observing receivers) is mathematically equivalent to session processing with all correlations only under certain conditions. This equivalence is verified empirically using simulated and real data. However, the conditions under which this equivalence holds are difficult to achieve in practice.

INTRODUCTION

Many software packages used for static GPS surveying are capable of only baseline processing, where individual baselines are processed separately. Consequently the mathematical correlations between simultaneously observed baselines are neglected. Moreover, all possible baselines, including so-called “trivial” baselines that are linear combinations of others, are often combined together in a 3D adjustment. The emphasis has been put on producing coordinate values with little regard to providing objective estimates of the accuracy or precision of the results. Realizing that many GPS users will have access to only baseline processing software, the objective of this study is to investigate processing strategies that could be used to provide results that are equivalent to session processing. These strategies may then be used to develop specifications for baseline processing of GPS data.

Because of subtle differences in GPS processing methods much of the terminology used in the literature can be confusing. In this paper we adopt the following definitions for the various GPS solutions and coordinate adjustments:

Baseline: Coordinate vector resulting from any station pair.
Session: An observing period of multiple receivers.
Independent baselines: A set of baselines where no individual baseline is a linear combination of any others.
Linearly dependent (trivial) baselines: Baselines which are linear combinations of others.
Baseline solution: Solution from processing a single baseline.

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Session adjustment or baseline processing: 3D least squares adjustment of all possible baselines solutions.
Session solution or session processing: Solution from simultaneous processing of all independent baselines with mathematical correlations between baselines.

In principle, only session solutions using all mathematical correlations are correct. This approach also allows for orbit improvement, spatial modeling of atmospheric effects and more reliable ambiguity resolution which leads to more realistic accuracy estimates. Covariance matrices obtained from such solutions preserve the correlations among the coordinates estimates thereby enabling proper integration of GPS results with other networks. On the other hand, baseline processing using all baseline combinations significantly distorts the formal accuracy estimates by artificially increasing the redundancy in the model. This results in overly optimistic covariance matrices even though the coordinates may be well determined.

To date there have been very few analyses that permit an objective evaluation of the effect of ignoring these correlations and including trivial baselines. Beutler et al. (1987), Beck et al. (1989), Hackman et al. (1989) and Hollmann et al. (1990) show there is little difference in the coordinate values when correlations are ignored and all baseline combinations are used. On the other hand, Vincenty (1987), Beck et al. (1989), Hollmann et al. (1990) and Jivall (1992) note that covariance matrices from such solutions are overly optimistic. Hollmann et al. (1990) attribute this to the effect of physical correlations. Vincenty (1987), Beck et al. (1989) and Jivall (1992), however, argue this is caused by including linearly dependent trivial baselines and recommend scaling the covariance matrix by a factor of n/2, where n is the number of simultaneously observing receivers, in order to compensate for this. Vincenty (1987) and Jivall (1992) also point out that all possible trivial baselines must be included to properly account for the neglect of the mathematical correlations.

As described in Jivall (1992), one would intuitively expect that, by including all baseline combinations, the covariance matrix should be multiplied by the factor n/2 to remove the artificial increase in the number of baselines. Including trivial baselines effectively increases the weight of the independent baselines by the factor n/2 (which comes from the total number of baseline combinations / number of independent baselines). However, except for the preliminary tests in Beck et al. (1989), there have been no empirical analyses of what the actual scale factor is in practice, how it varies and under what conditions it holds. We therefore examine, both theoretically and empirically, the effects of neglecting correlations between baselines and using trivial baselines.

In all of our analyses we have estimated all carrier phase ambiguities to avoid comparing solutions with different ambiguities resolved. In general we found that ambiguity resolution was more difficult in baseline processing than session processing.

The problem of unrealistic covariance matrices due to incomplete modeling of all the (physical) effects on GPS observations is not discussed here. Such effects produce so-called physical correlations among the observations. These correlations tend to be positive, thereby reducing the mathematically induced correlations.

EQUIVALENCE OF PROCESSING METHODS

The mathematical equivalence of session and baseline processing can be shown by examining the normal equations for each type of solution. Although we use double difference carrier phase observations in the following developments, the results are independent of the differencing method used since all can be shown to be mathematically equivalent to each other using the fundamental differencing theorem (see Lindlohr and Wells, 1985).

Consider a single epoch with n receivers and let $\mathbf{C}_\Phi = \mathrm{diag}(\sigma_\Phi{}^2)$ denote the a priori covariance matrix (uncorrelated) of the carrier phase observations Φ. The single differences observations $\nabla\Phi_i$ and their fully populated covariance matrix $\mathbf{C}_{\nabla\Phi_i}$ at a single receiver i are then

$$\nabla\Phi_i = \mathbf{S}\,\Phi_i\,,$$
$$\mathbf{C}_{\nabla\Phi_i} = \mathbf{S}\,\mathbf{C}_{\Phi_i}\,\mathbf{S}^T = \sigma_\Phi{}^2\,\mathbf{S}\,\mathbf{S}^T\,.$$

where $\mathbf{S}$ is the between-satellite single difference operator and the covariance matrix is the same at each receiver. Similarly, for the session solution the linearly independent double differences $\Delta\nabla\Phi_I$ and their covariance matrix $\mathbf{C}_S$ are given by

$$\Delta\nabla\Phi_I = \mathbf{D}_I\,\nabla\Phi\,,$$
$$\mathbf{C}_S = \mathbf{D}_I\,\mathbf{C}_{\nabla\Phi}\,\mathbf{D}_I{}^T\,.$$

where $\mathbf{D}_I$ is the double difference operator that generates a linearly independent set of double differences from the single differences and $\mathbf{C}_{\nabla\Phi} = \mathrm{diag}(\mathbf{C}_{\nabla\Phi 1}, \mathbf{C}_{\nabla\Phi 2}, ..., \mathbf{C}_{\nabla\Phi n})$ is the single difference covariance matrix for all n receivers. Note that $\mathbf{C}_S$ is fully populated and accounts for the mathematical correlations between receivers introduced by the double differencing process.

The least squares parameter estimates δ_S (coordinates and possibly ambiguities) and their covariance matrix $\mathbf{C}_{\delta S}$ for the session solution can be written in terms of the set of linearly independent double differences as

$$\delta_S = \mathbf{N}_S{}^{-1}\,\mathbf{u}_S\,,$$
$$\mathbf{C}_{\delta S} = \mathbf{N}_S{}^{-1}\,,$$

where $\mathbf{N}_S$ is the normal equation matrix and $\mathbf{u}_S$ is the so-called constant vector. (For a review of least squares theory, see Vaníček and Krakiwsky, 1986). Letting $\mathbf{A}_I$

denote the design matrix relating the double differences to the parameters, we get

$$\mathbf{N}_S = \mathbf{A}_I^T \mathbf{P}_S \mathbf{A}_I ,$$
$$\mathbf{u}_S = \mathbf{A}_I^T \mathbf{P}_S \Delta\nabla\Phi_I ,$$

where $\mathbf{P}_S = \mathbf{C}_S^{-1}$ is the correct (fully populated) weight matrix for the double differences.

In baseline processing, all individual baselines can be either separately processed (baseline solution) and subsequently combined in a 3D session adjustment or processed together as in session processing but without the correlations between baselines. These approaches are equivalent only when all integer ambiguities are identically resolved. We use the latter since it is easier to show the equivalence with session processing.

Letting $\mathbf{A}_B$ denote the design matrix relating baseline double differences $\Delta\nabla\Phi_B$ to the parameters, the normal equation matrix $\mathbf{N}_B$ and constant vector $\mathbf{u}_B$ are given by

$$\delta_B = \mathbf{N}_B^{-1} \mathbf{u}_B ,$$
$$\mathbf{C}_{\delta B} = \mathbf{N}_B^{-1} ,$$

where

$$\mathbf{N}_B = \mathbf{A}_B^T \mathbf{P}_B \mathbf{A}_B ,$$
$$\mathbf{u}_B = \mathbf{A}_B^T \mathbf{P}_B \Delta\nabla\Phi_B ,$$

and $\mathbf{P}_B = \mathbf{C}_B^{-1} = \mathrm{diag}(2\mathbf{C}_{\nabla\Phi 1}, 2\mathbf{C}_{\nabla\Phi 2}, ..., 2\mathbf{C}_{\nabla\Phi n})^{-1}$ is the weight matrix for all baseline double differences (uncorrelated between baselines).

Partitioning all possible baseline combinations into linearly independent (subscript I) and dependent (subscript T) baseline sets, we can rewrite the normal equations as

$$\mathbf{N}_B = \mathbf{A}_I^T \mathbf{P}_I \mathbf{A}_I + \mathbf{A}_T^T \mathbf{P}_T \mathbf{A}_T$$
$$\mathbf{u}_B = \mathbf{A}_I^T \mathbf{P}_I \Delta\nabla\Phi_I + \mathbf{A}_T^T \mathbf{P}_T \Delta\nabla\Phi_T$$

where $\mathbf{P}_I$ and $\mathbf{P}_T$ are both block diagonal matrices of the form $\mathrm{diag}(2\mathbf{C}_{\nabla\Phi 1}, 2\mathbf{C}_{\nabla\Phi 2}, ...)^{-1}$. Realizing that the trivial baselines are linear combinations of the independent ones, we can also write

$$\Delta\nabla\Phi_T = \mathbf{T}\Delta\nabla\Phi_I ,$$
$$\mathbf{A}_T = \mathbf{T}\,\mathbf{A}_I ,$$

where $\mathbf{T}$ is the matrix transforming the independent baselines into trivial ones. The normal equations can now be expressed as

$$\mathbf{N}_B = \mathbf{A}_I^T \mathbf{P}_B^* \mathbf{A}_I ,$$
$$\mathbf{u}_B = \mathbf{A}_I^T \mathbf{P}_B^* \Delta\nabla\Phi_I ,$$

where $\mathbf{P}_B^* = (\mathbf{P}_I + \mathbf{T}^T \mathbf{P}_T \mathbf{T})$.

To show the equivalence of the two approaches we need to show when $\mathbf{P}_S$ and $\mathbf{P}_B^*$ are equivalent. For example consider, without loss of generality, taking double differences with respect to the first receiver in a session. The double differencing operator for the independent session baselines will have the form

$$\mathbf{D}_S = \begin{bmatrix} -\mathbf{I} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ -\mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} \end{bmatrix} .$$

The resulting double difference covariance matrix for the session solution of independent baselines is then

$$\mathbf{C}_S = \begin{bmatrix} 2\mathbf{C}_{\nabla\Phi i} & \mathbf{C}_{\nabla\Phi i} & \cdots & \mathbf{C}_{\nabla\Phi i} \\ \mathbf{C}_{\nabla\Phi i} & 2\mathbf{C}_{\nabla\Phi i} & \cdots & \mathbf{C}_{\nabla\Phi i} \\ \vdots & \vdots & & \vdots \\ \mathbf{C}_{\nabla\Phi i} & \mathbf{C}_{\nabla\Phi i} & \cdots & 2\mathbf{C}_{\nabla\Phi i} \end{bmatrix}$$

and the weight matrix is

$$\mathbf{P}_S = \begin{bmatrix} \frac{n-1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} & -\frac{1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} & \cdots & -\frac{1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} \\ -\frac{1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} & \frac{n-1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} & \cdots & -\frac{1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} \\ \vdots & \vdots & & \vdots \\ -\frac{1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} & -\frac{1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} & \cdots & \frac{n-1}{n}\mathbf{C}_{\nabla\Phi i}^{-1} \end{bmatrix} .$$

For baseline processing, we have the double difference weight matrix

$$\mathbf{P}_B^* = \mathbf{P}_I + \mathbf{T}^T \mathbf{P}_T \mathbf{T} ,$$

where the form of $\mathbf{T}$ depends on the number of receivers as well as $\mathbf{D}_S$. For 4 receivers and the differencing scheme above,

$$\mathbf{T} = \begin{bmatrix} -\mathbf{I} & \mathbf{I} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & -\mathbf{I} & \mathbf{I} \end{bmatrix}$$

The double difference weight matrix is then

$$\mathbf{P}_B^* = \begin{bmatrix} \frac{n-1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} & -\frac{1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} & \cdots & -\frac{1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} \\ -\frac{1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} & \frac{n-1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} & \cdots & -\frac{1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} \\ \vdots & \vdots & & \vdots \\ -\frac{1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} & -\frac{1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} & \cdots & \frac{n-1}{2}\mathbf{C}_{\nabla\Phi i}^{-1} \end{bmatrix}$$

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Because the form of $\mathbf{T}$ depends on the differencing scheme and the numbers of receivers, we numerically verified the above form of $\mathbf{P}_B$ for different differencing schemes and from 3 to 8 receivers.

From an inspection of the two weight matrices we find that $\mathbf{P}_S = (2/n)\mathbf{P}_B^*$. The normal equations for both the session and baseline solution are therefore identical when the factor 2/n is included in $\mathbf{P}_B^*$ and thus

$$\delta_S = \delta_B ,$$
$$\mathbf{C}_{\delta S} = \frac{n}{2}\mathbf{C}_{\delta B} .$$

For the mathematical equivalence of session and baseline processing to hold, the following conditions must be satisfied for baseline processing:

- All possible linear combinations must be processed as baseline solutions, either individually or together in a session solution without correlations between baselines.
- The trivial baselines must be exact linear combinations of the independent baselines (i.e., all of the same data is used).
- All possible baseline solutions must be subsequently combined together in a session adjustment. The covariance matrix for each baseline solution must be scaled by the appropriate n/2 factor Note that n will be different for sessions using different numbers of receivers. If the same number of receivers is used in all sessions, only the final network covariance matrix needs to be scaled by n/2.
- Care must be exercised in the handling of the variance factor from the individual baseline solutions. Because the variance factor is derived from random quantities (residuals), it will vary slightly from baseline to baseline which could adversely affect the relative weighting of baselines in the session adjustment. To maintain equivalence with the session solution, the same variance factor should be used for all baseline solutions; e.g., a "pooled" variance factor derived from those for each baseline – i.e., work back to the sum of squared residuals, accumulate these for all baselines and divide by the total degrees of freedom for each session. Note, however, that the estimated variance factor determined from the 3D adjustment of the individual baseline coordinates must not be applied.

Note that the mathematical equivalence holds only when the ambiguities are identically resolved.

TESTS WITH SIMULATED DATA

The equivalence of session and baseline processing was tested empirically using simulated GPS observations of an actual survey of the Ottawa GPS basenet. The relative location of the 8 stations in the network is illustrated in Figure 1. The baseline lengths ranged from 2 to 150 km.

The simulated data for this network were generated using the Bernese GPS Software v3.3. Phase observations were computed for a single session at each of the 8 stations

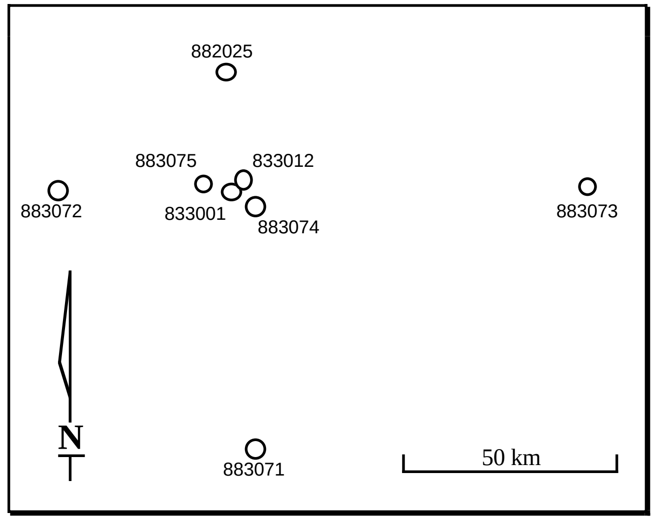

Figure 1: Ottawa GPS basenet.

using a standard deviation of 3 mm and exactly the same start/stop times. The duration of the session was 6 hours. A broadcast ephemeris obtained from an actual GPS survey of the same network on day 337 (December 3), 1991 was used for the simulation. No cycle slips or atmospheric effects were introduced.

Using this data, session solutions were obtained using station 833001 as the reference receiver for generating the 7 linearly independent baselines. Individual baseline solutions were then computed for all possible (28) baseline combinations. After removing the variance factors from each baseline solution, they were combined in a 3D adjustment (session adjustment). The variance factors where also removed from the session solution (for comparison purposes only). In both solutions, all double difference phase ambiguities were estimated. Similar session adjustments of baseline solutions were also computed for different numbers of these stations in order to check the variation of the n/2 scale factor with the number of receivers.

The differences between the coordinate estimates for session and baseline processing are summarized in Table 1. Clearly the differences between the solutions are only on the sub-millimeter level as expected for simulated data. The largest (0.65 mm) occur for the case with 5 receivers.

The covariance matrices are compared in Figure 2 which depicts the variations in the ratios of all the covariance matrix elements between session and baseline processing. The ratio should be equivalent to the scale factor n/2=4. The means of the ratios agree well with the theoretical n/2 value in all cases except those for 3 and 5 receivers. We also observe small variations in the ratios for each comparison. The differences are attributed to differences in the estimated ambiguities.

We also investigated the effect of omitting one and two baselines from the set of all possible combinations, as is often done in practice when "outlier" baselines are detected in the 3D adjustment. It is important to point

Table 1: Comparison of coordinate estimates (session–baseline) for different numbers of receivers and simulated data. Fixed station was removed from comparison.

8 Receivers			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
833012	0.04	0.01	0.00
882025	-0.04	0.15	0.01
883071	-0.05	0.13	-0.00
883072	-0.06	0.06	-0.07
883073	-0.00	0.09	0.09
883074	-0.05	0.13	-0.01
883075	-0.07	0.10	-0.03
	Mean 0.02	St.Dev. 0.07	
7 Receivers			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
882025	-0.06	0.15	0.01
883071	-0.07	0.03	0.04
883072	-0.05	0.04	-0.09
883073	-0.06	0.05	0.07
883074	-0.08	0.12	-0.01
883075	-0.11	0.09	-0.03
	Mean 0.00	St.Dev. 0.08	
6 Receivers			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
883071	-0.07	-0.26	0.19
883072	-0.10	-0.26	0.06
883073	-0.08	-0.20	0.26
883074	-0.10	-0.13	0.19
883075	-0.16	-0.18	0.14
	Mean -0.05	St.Dev. 0.17	
5 Receivers			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
883072	-0.10	0.52	-0.32
883073	-0.13	0.65	-0.19
883074	-0.16	0.62	-0.21
883075	-0.21	0.56	-0.25
	Mean 0.07	St.Dev. 0.39	
4 Receivers			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
883073	-0.24	0.07	0.27
883074	-0.28	0.04	0.24
883075	-0.28	-0.03	0.18
	Mean 0.00	St.Dev. 0.22	
3 Receivers			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
883074	0.26	0.32	-0.03
883075	0.25	0.18	-0.03
	Mean 0.16	St.Dev. 0.15	

Table 2: Comparison of coordinate estimates (session–baseline) for simulated data with 1 and 2 baselines omitted from baseline solutions. Fixed station was removed from comparison.

5 Receivers – No baseline omitted			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
883072	-0.10	0.52	-0.32
883073	-0.13	0.65	-0.19
883074	-0.16	0.62	-0.21
883075	-0.21	0.56	-0.25
	Mean 0.07	St.Dev. 0.39	
5 Receivers – 1 baseline omitted			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
883072	-0.10	0.52	-0.32
883073	-0.13	0.65	-0.19
883074	-0.09	0.62	-0.23
883075	-0.28	0.57	-0.23
	Mean 0.07	St.Dev. 0.39	
5 Receivers – 2 baselines omitted			
Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
883072	-0.10	0.52	-0.32
883073	-0.05	0.67	-0.19
883074	-0.24	0.60	-0.21
883075	-0.25	0.63	-0.33
	Mean 0.06	St.Dev. 0.41	

out, however, that omitting any of the baselines invalidates any equivalence with session processing. The results of the comparison of session processing (with all independent baselines) and baseline processing (with one and two baselines omitted) are summarized in Table 2 and Figure 3.

As expected with simulated data, the coordinates are practically unaffected. On other hand, the ratios of the covariance matrix elements are shifted further from the expected theoretical n/2 value and also display much larger variations – as much as a factor of 4 when two baselines are omitted! Clearly the covariance matrix is greatly affected by the omission of baselines and therefore this should always be avoided.

TESTS WITH REAL DATA

Tests were also performed with real GPS data collected on the same Ottawa GPS basenet but using only 5 stations. The survey was performed in 1991 by the Canadian Mapping and Charting Establishment. One 6 hour session was observed on each of days 337 (December 3) to 340 (December 6). Five new Ashtech P12 P-code receivers were used on all days except the first (when only 4 were used).

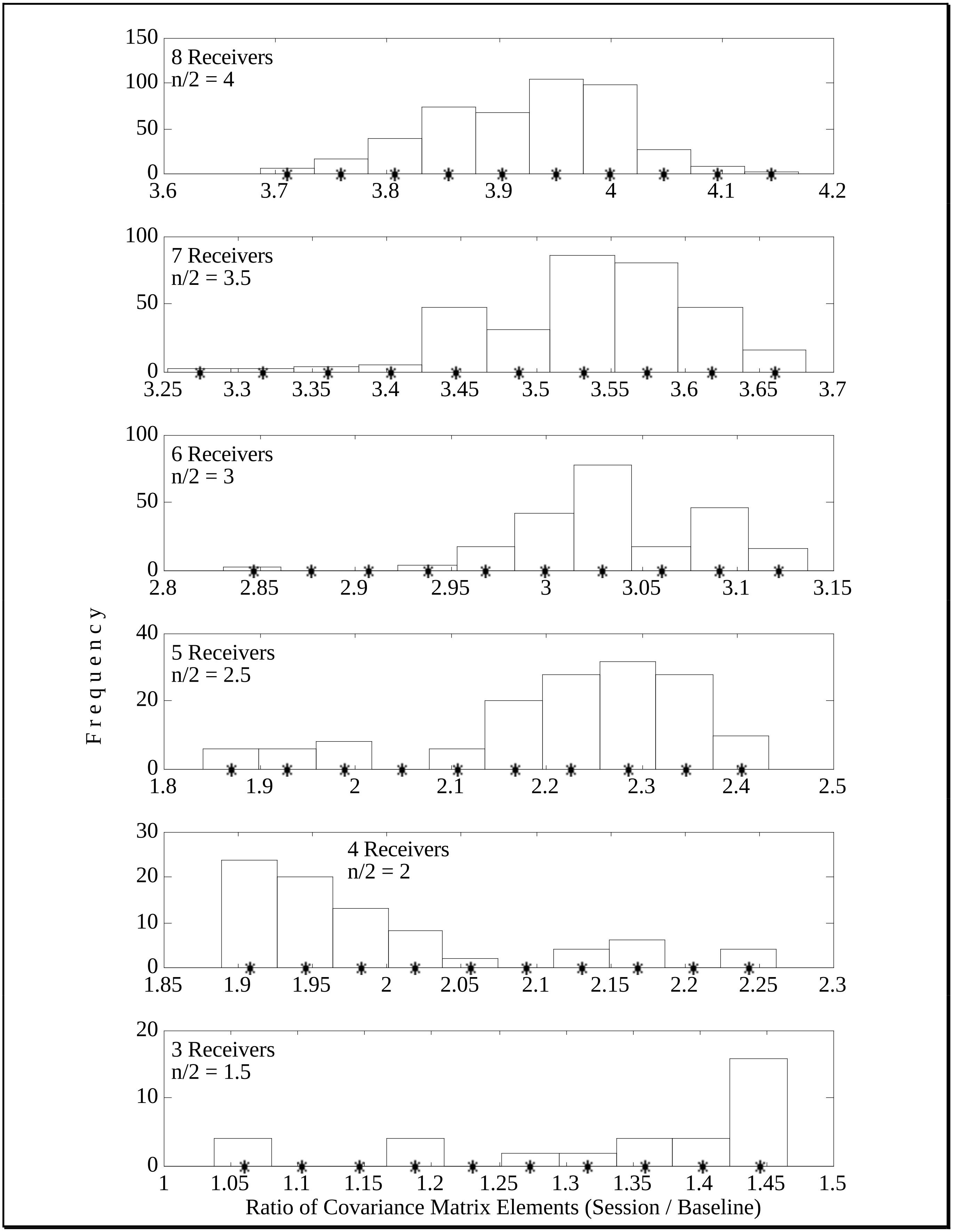

Figure 2: Frequency distributions of ratios of covariance matrix elements (session/baseline) for simulated data.

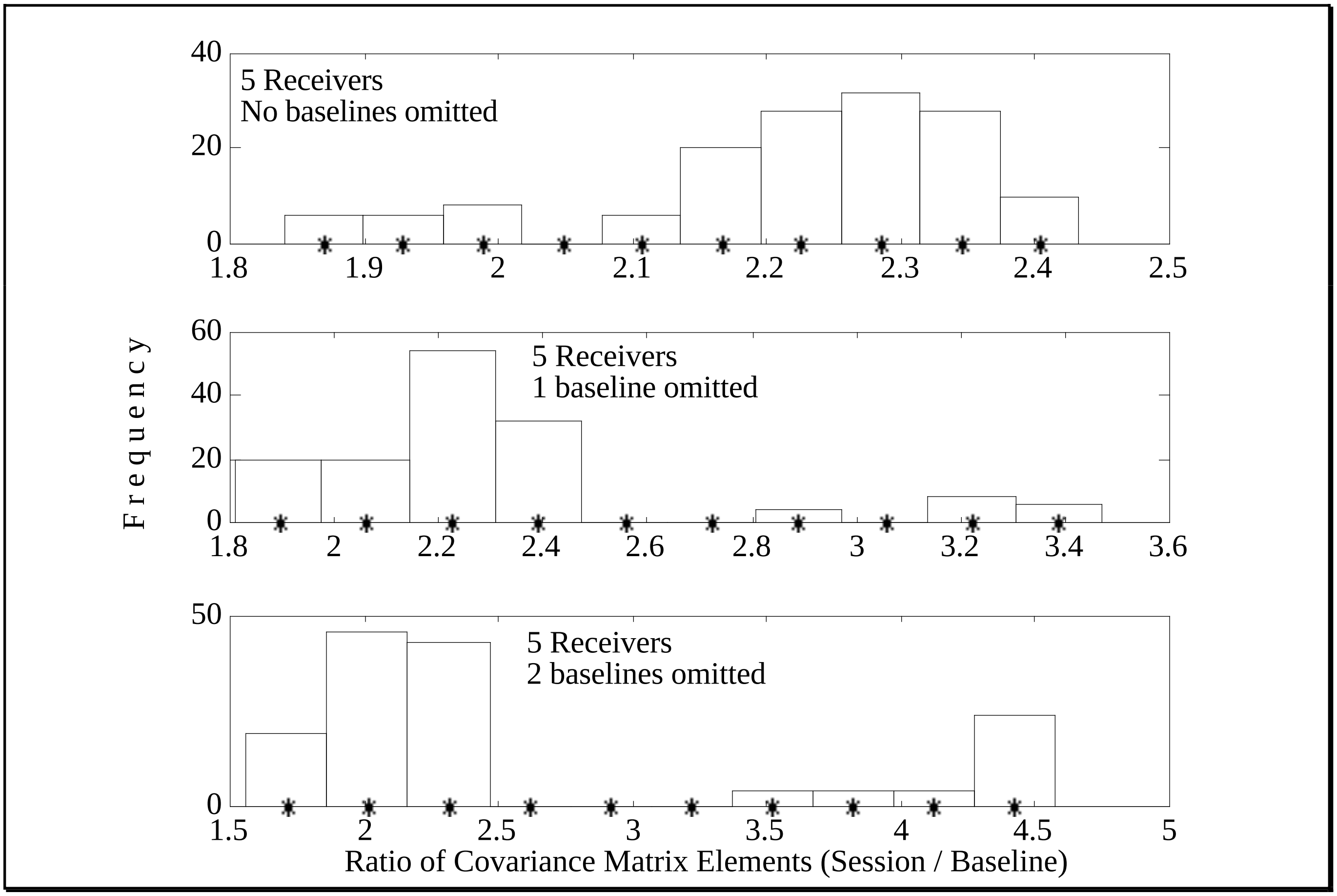

Figure 3: Frequency distributions of ratios of covariance matrix elements (session/baseline) for simulated data with 1 and 2 baselines omitted from baseline solutions.

The data were processed using the Bernese GPS software with the same fixed station, processing options and procedures that were used for the simulations. However, not all of the five receivers started and stopped recording data at exactly the same time. At three stations, about 15-20 minutes of additional data was collected. Session and baseline solutions were computed for each day. All carrier phase ambiguities were estimated.

The differences between the coordinate estimates from session and baseline processing are summarized in Table 3. Clearly, the differences are much larger than for the simulated data with the largest being 1 to 2 cm. Nevertheless, the mean of the discrepancies is practically zero (less than 0.2 mm) for all days. These discrepancies are attributed to differences in the estimated ambiguities.

The covariance matrices are compared in Figure 4, again in terms of variations in the ratios of the covariance matrix elements for session and baseline processing. The means of the ratios agree very well with the theoretical n/2 value for all days except 337 (with 4 receivers instead of 5). Similar to our simulations, we again see small variations in the ratios for each comparison. These variations are also attributed to differences in the estimated ambiguities.

We also investigated the effect of omitting one and two baselines from the set of all possible combinations in the 3D adjustment. We emphasize that omitting any of the baselines invalidates any equivalence with session processing. The results of the comparison of session processing (with all independent baselines) and baseline processing (with one and two baselines omitted) are summarized in Table 4 and Figure 5. This small sample shows effects up to 7 mm, which would vary with respect to baseline removed, baseline length, conditions, etc. In addition, the session/baseline ratios of the covariance matrix elements again exhibited many values 4 to 5.5 times larger than the theoretical one! This caused the mean to be shifted further from the theoretical value. Clearly the covariance matrix is adversely affected by the omission of baselines.

Although solutions were also computed with ambiguities fixed, many more ambiguities were unresolved in the baseline processing mode than for session processing. This resulted in large differences between the solutions which overwhelmed any due to the baseline processing. For this reason we didn't make any comparisons among these solutions.

Table 3: Comparison of coordinate estimates (session–baseline) for real data. Fixed station was removed from comparison.

Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
Day 337 – 4 Receivers			
882025	6.23	-19.79	7.62
883072	11.45	1.87	0.72
883073	-11.73	-11.63	16.99
Mean 0.19 St.Dev. 12.17			
Day 338 – 5 Receivers			
882025	5.74	-17.53	3.87
883072	9.39	7.07	-5.20
883073	-13.39	-3.57	7.39
883074	-1.38	12.33	-6.69
Mean -0.17 St.Dev. 9.37			
Day 339 – 5 Receivers			
882025	6.29	-14.77	2.44
883072	12.36	7.52	-4.46
883073	-20.39	-7.57	6.39
883074	2.21	13.33	-3.92
Mean -0.05 St.Dev. 10.47			
Day 340 – 5 Receivers			
882025	5.72	-17.66	3.90
883072	10.05	7.54	-6.17
883073	-13.76	-3.69	7.93
883074	-1.22	12.79	-7.13
Mean -0.14 St.Dev. 9.71			

CONCLUSIONS AND RECOMMENDATIONS

It has been shown that session and baseline processing of GPS data are mathematically equivalent when the covariance matrix for baseline processing is scaled by the factor n/2, where n is the number of receivers used in any observing session. The requirements for achieving this equivalence are that (i) all possible baselines (including trivial ones) be used in a 3D session adjustment, (ii) the same variance factor (such as a "pooled" estimate) must be used in each individual baseline solution, (iii) the trivial baselines must be exact linear combinations of the independent baselines, and (iv) identical integer ambiguities are resolved in both session and baseline solutions.

These conditions for equivalence of session and baseline processing (especially identical ambiguity resolution) can be difficult to achieve in practice. Nevertheless, the results show that an essentially equivalent covariance matrix can be determined from baseline processing,

Table 4: Comparison of coordinate estimates (session–baseline) for real data with 1 and 2 baselines omitted from baseline solutions. Fixed station was removed from comparison.

Station	ΔX (mm)	ΔY (mm)	ΔZ (mm)
Day 338 – 5 Receivers – No baselines omitted			
882025	5.74	-17.53	3.87
883072	9.38	7.07	-5.20
883073	-13.39	-3.57	7.39
883074	-1.38	12.33	-6.69
Mean -0.17 St.Dev. 9.37			
Day 338 – 5 Receivers – 1 baseline omitted			
882025	5.74	-17.53	3.87
883072	9.38	7.07	-5.20
883073	-11.89	-0.25	4.40
883074	-2.82	9.31	-3.90
Mean -0.15 St.Dev. 8.48			
Day 338 – 5 Receivers – 2 baselines omitted			
882025	5.74	-17.56	3.85
883072	2.14	7.51	-6.07
883073	-8.85	-3.10	7.92
883074	1.40	12.02	-6.39
Mean -0.12 St.Dev. 8.48			

although small coordinate differences of up to 5 mm can result. For many applications this would be significant.

When using baseline processing, care must be taken to ensure that the covariance matrices are scaled by the proper n/2 factor. When different numbers of receivers are used in different sessions, the baselines from each session will need to be scaled accordingly before combining them in a session adjustment.

Care must also be exercised when dealing with the different variance factors determined from the GPS baseline solutions and the 3D session adjustment. To achieve equivalence of the covariance matrices, a variance factor compatible with that used for the session solution should be used (e.g., a "pooled" estimate). The variance factor from the 3D session adjustments of the individual baseline solutions should never be used.

Although we have shown that session and baseline processing are equivalent under certain conditions, session processing has its advantages. In particular it allows for orbit improvement, spatial modeling of atmospheric effects, and enables easier ambiguity resolution. On the other hand, a problem with baseline processing is that some baselines may not have enough data available to resolve the ambiguities, especially on longer lines.

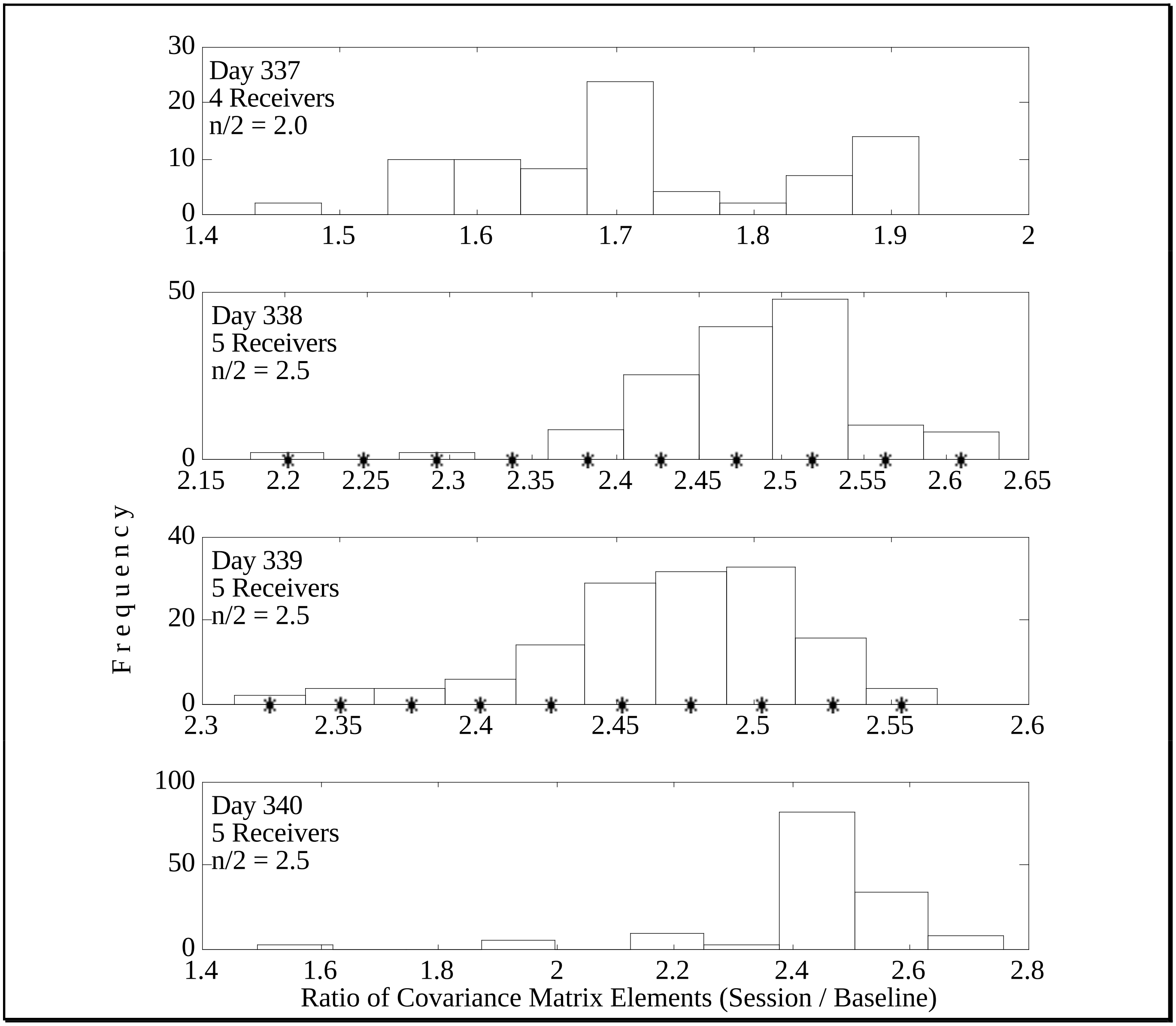

Figure 4: Frequency distribution of ratios of covariance matrix elements (session/baseline) for real data.

It is important to emphasize that all possible baselines, including linearly dependent (trivial) ones, must be used in the 3D adjustment of the independent baseline solutions to maintain equivalence with session processing. We have found that omitting a baseline adversely affects both the coordinates and their covariance matrix. If, in a 3D session adjustment of all baseline solutions, one of the baselines is considered to be a outlier, then the data should be inspected on the receiver level. If it is found that part of the data is bad, then all baselines using the receiver should be reprocessed without this data. If for some reason they can't be reprocessed, then all baselines using the receiver should be omitted from the session adjustment. This would then be equivalent to session processing.

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DISCLAIMER

No product endorsement is herein intended or implied.

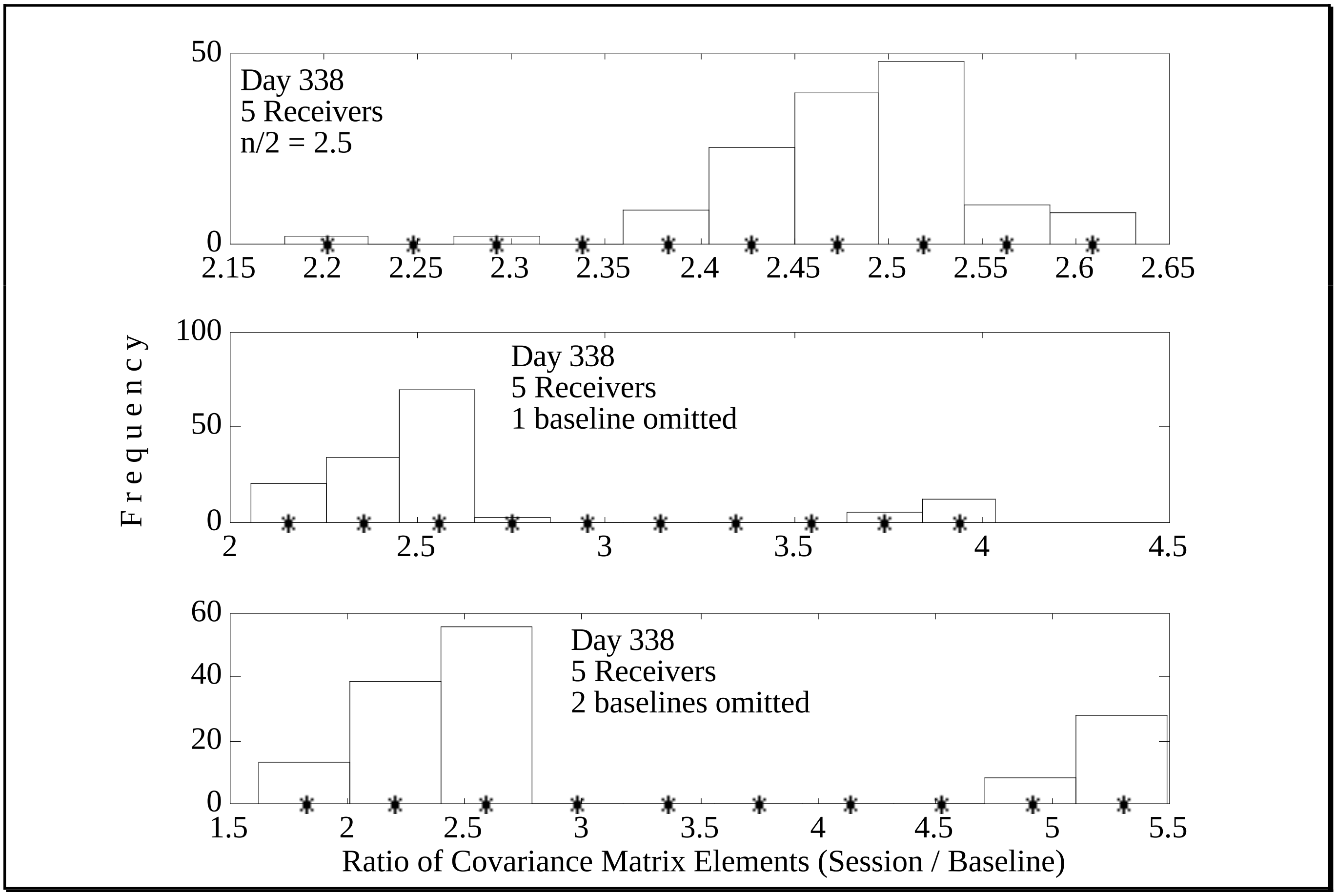

Figure 5: Frequency distributions of ratios of covariance matrix elements (session/baseline) for real data with 1 and 2 baselines omitted from baseline solutions.

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