Algorithms for Azimuth Determination

by

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ABSTRACT

This paper provides the expressions necessary to determine astronomic azimuths from observations on the sun or polaris without recourse to astronomical tables. No attempt is made here to define concepts or theory. It is merely intended to provide only the mathematical expressions that may be programmed into a computer or calculator for azimuth determination without reference to any astronomical tables. For a detailed treatment of the theory see Craymer [1984].

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Addendum and Errata

1. Accuracy of Algorithms

The algorithms presented in this paper are intended to provide accuracies of the order of 1 second of arc in both right ascension and declination. Reference should be made to Craymer (1984) for additional terms in many of the equations if greater accuracy is desired.

Furthermore, these algorithms are based upon the FK4 Star Catalogue and the 1964 system of astronomical constants. In 1984 a new system of constants and theory of notation superseded the old one. The effective difference between the two systems may be neglected, however, for the accuracies proposed here.

2. §3.3 Removal of the Elliptical Aberration Correction from the Catalogued Coordinates

The FK4 Star Catalogue lists the right ascension and declination of stars with the corrections for the effect of elliptical aberration already applied. For precise updates (i.e., for accuracies of better than 0.2 seconds of time), this correction should be removed from the catalogued values prior to updating. The correction, computed for the date of observation, must then be re-applied after the update (see §3.8).

If this high accuracy is not required, sections 3.3 and 3.8 may be omitted. The accuracies in this case will be of the order of the difference between the corrections for the catalogue and observation epochs (e.g., about 0.2 seconds of time over a period of about twenty-five years).

On the other hand, if the elliptical aberration correction is to be removed, the following values of right ascension and declination should be used in place of the catalogued ones (i.e., neglect the equations in section 3.3):

α_0 = right ascension of polaris for 1975.0 uncorrected for annual elliptical aberration
= 31.858302 (deg.)

δ_0 = declination of polaris for 1975.0 uncorrected for annual elliptical aberration
= 89.149867 (deg.)

These values should be used for the proper motion correction in section 3.4.

3. §3.7 Annual Circular Aberration Correction

In the equation for $\Delta\delta_A$, replace $\tan\varepsilon$ ($\tan\delta_N$) with $\tan\varepsilon \cos\delta_N$.

4 §3.8 Annual Elliptical Aberration Correction

The equations given in this section should only be used when the elliptical aberration corrections have been removed from the catalogued coordinates.

There is an error in the equations for $\Delta\alpha_E$ and $\Delta\delta_E$. The constant coefficient in both equations should be Δ0.343 instead of -20.496.

5. Errors in First Version of the Paper

§1.2 Time Intervals - ignore the equation for T_m; it should not be used anywhere in the paper.

§1.6 Greenwich Apparent Sidereal Time - replace T_m with T_e in the equation for GMST.

6. Acknowledgements

We would like to acknowledge Mr. Steve Balaban for his painstaking testing of these algorithms while on assignment in Libya. His efforts have revealed most of the errors reported here. Interestingly, the results of his testing have been better than 0.02 seconds of time in right ascension and 0.1 seconds of arc in declination.

7. §1.6 : GMST = UT + 6.646066 + ...

§3.1 : α_c = 2.1240350 (hr).

TABLE OF CONTENTS

1. Preliminary Data

The following parameters must be computed before updating the astronomic coordinates of both the sun and polaris.

1.1 Julian Dates

JD (1900, Jan $0^{d}.5$ UT) = 2415020.0 (days)

JD (1975.0) = 2442413.478 (days)

JD (for Year=Y, Month=M, day=D, Universal Time=UT)

$$= INT(365.25\ Y) + INT(30.6001(M+1)) + D + UT/24 + B + 1720994.5,$$

where

$$A = INT(Y/100)$$

$$B = 2 - A + INT(A/4)$$

and

$$\text{if } M \leq 2,\ Y = Y - 1$$

$$M = M + 12$$

Note that INT indicates the truncated integer value (i.e., INT(3.291) = 3).

1.2 Time Intervals

T_e = interval of ephemeris centuries elapsed since 1900, Jan. $0^{d}.5$ UT

= (JD − 2415020.0) / 36525 (ephemeris centuries)

1.3 Orbital Parameters

M_s = mean anomaly of sun

$= 358.475833 + 35999.04975\ T_e - 0.00015\ T_e^2 -$

$- 0.000003\ T_e^3$ (deg.)

M_{mn} = mean anomaly of moon

$= 296.104608 + 477198.849108\ T_e + 0.009192\ T_e^2 +$

$+ 0.000014\ T_e^3$ (deg.)

M_v = mean anomaly of venus

$= 212.603222 + 58517.803875\ T_e + 0.001286\ T_e^2$ (deg.)

M_m = mean anomaly of mars

$= 319.529022 + 19139.859219\ T_e + 0.000181\ T_e^2$ (deg.)

M_j = mean anomaly of jupiter

$= 225.32833 + 3034.96202\ T_e - 0.000722\ T_e^2$ (deg.)

M_{sn} = mean anomaly of saturn

$= 175.46622 + 1221.55147\ T_e - 0.000502\ T_e^2$ (deg.)

D = mean elongation of moon from sun

$= 350.737486 + 445267.114217\ T_e - 0.001436\ T_e^2$ (deg.)

F = mean argument of ecliptic latitude of moon

$= 11.250889 + 483202.02515\ T_e - 0.003211\ T_e^2$ (deg.)

Ω = longitude of mean ascending node of moon

$= 259.183275 - 1934.142008\ T_e + 0.002078\ T_e^2$ (deg.)

ω = mean longitude of perihelion

$= 101.220833 + 1.719175\ T_e + 0.000453\ T_e^2$ (deg.)

ω' = mean longitude of perigee

$= \omega + 180$ (deg.)

1.3 Orbital Parameters (cont'd)

e = eccentricity of earth's orbit

$$= 0.01675104 - 0.00004180\ T_e - 0.00000013\ T_e^2$$

1.4 Nutation Parameters

$\Delta\lambda_N$ = nutation in ecliptic longitude

$$= -17.233 \sin\Omega + 0.209 \sin 2\Omega - 1.273 \sin(2\Omega - 2D + 2F) +$$

$$+ \ 0.126 \sin M_S - 0.204 \sin(2\Omega + 2F) \ \text{(arcsec.)}$$

$\Delta\varepsilon_N$ = nutation in obliquity

$$= \ 9.210 \cos\Omega - 0.090 \cos 2\Omega + 0.552 \cos(2\Omega - 2D + 2F) +$$

$$+ \ 0.088 \cos(2\Omega + 2F) \ \text{(arcsec.)}$$

1.5 Obliquity of the Ecliptic

$$\varepsilon = \overline{\varepsilon} + \Delta\varepsilon_N,$$

where

$\overline{\varepsilon}$ = mean obliquity of the ecliptic

$$= 23.452294 - 0.013013\ T_e - 0.000002\ T_e^2 \ \text{(deg.)}$$

1.6 Greenwich Apparent Sidereal Time

GAST = Greenwich apparent sidereal time

$$= \text{GMST} + \Delta\lambda_N \cos\varepsilon,$$

where

GMST = Greenwich mean sidereal time

$$= \text{UT} + 6.6460\overset{6}{\not{0}}6 + 2400.051262\ T_e + 0.000026\ T_e^2 \ \text{(hr.)}$$

$\Delta\lambda_N$ is in hours

2. Solar Update

The following sequence of expressions were derived by Newcomb [1898]. These give the apparent astronomic coordinates (α, δ) of the sun.

2.1 True Ecliptic Longitude of Sun

λ = true (geometic) longitude of the sun

$$= L + C + \Delta\lambda_{LP} + \Delta\lambda_{mn} + \Delta\lambda_P + \Delta\lambda_N ,$$

where

L = mean longitude of the sun

$$= 279.696678 + 36000.768925\ T_e + 0.000303\ T_e^2 \quad \text{(deg.)}$$

C = equation of the centre

$$= (1.9194603 - 0.0047889\ T_e - 0.0000144\ T_e^2)\ \sin M_s +$$
$$+ (0.0200939 - 0.0001003\ T_e)\ \sin 2M_s +$$
$$+ 0.0002928\ \sin 3M_s + 0.0000050\ \sin 4M_s \quad \text{(deg.)}$$

$\Delta\lambda_{LP}$ = long period perturbations in longitude

$$= 6.40\ \sin(231^{\circ}.19 + 20^{\circ}.20\ T_e) +$$
$$+ 0.266\ \sin(31^{\circ}.80 + 119^{\circ}.00\ T_e) +$$
$$+ 1.882\ \sin(57^{\circ}.24 + 150^{\circ}.27\ T_e) +$$
$$+ 0.202\ \sin(315^{\circ}.60 + 893^{\circ}.30\ T_e) \quad \text{(arcsec.)}$$

$\Delta\lambda_{mn}$ = lunar perturbation in longitude

$$= 6.454\ \sin D + 0.013\ \sin 3D + 0.177\ \sin(D+M_{mn}) -$$
$$- 0.424\ \sin(D-M_{mn}) + 0.039\ \sin(3D-M_{mn}) -$$
$$- 0.064\ \sin(D+M_s) + 0.172\ \sin(D-M_s) -$$
$$- 0.013\ \sin(D-M_{mn} - M_s) \quad \text{(arcsec.)}$$

2.1 True Ecliptic Longitude of Sun (cont'd)

$\Delta\lambda_p$ = planetary perturbation in longitude

$$= 4.838 \cos(299^{\circ}.102+M_V-M_S) + 0.116 \cos(148^{\circ}.900+2M_V-M_S) +$$
$$+ 5.526 \cos(148^{\circ}.313+2M_V-2M_S) + 2.497 \cos(315^{\circ}.943+2M_V-3M_S) +$$
$$+ 0.666 \cos(177^{\circ}.710+3M_V-3M_S) + 1.559 \cos(345^{\circ}.253+3M_V-4M_S) +$$
$$+ 1.024 \cos(318^{\circ}.150+3M_V-5M_S) + 0.210 \cos(206^{\circ}.200+4M_V-4M_S) +$$
$$+ 0.144 \cos(195^{\circ}.400+4M_V-5M_S) + 0.152 \cos(343^{\circ}.800+4M_V-6M_S) +$$
$$+ 0.123 \cos(195^{\circ}.300+5M_V-7M_S) + 0.154 \cos(359^{\circ}.600+5M_V-8M_S) +$$
$$+ 0.273 \cos(217^{\circ}.700-M_M+M_S) + 2.043 \cos(343^{\circ}.888-2M_M+2M_S) +$$
$$+ 1.770 \cos(200^{\circ}.402-2M_M+M_S) + 0.129 \cos(294^{\circ}.200-3M_M+3M_S) +$$
$$+ 0.425 \cos(338^{\circ}.880-3M_M+2M_S) + 0.500 \cos(105^{\circ}.180-4M_M+3M_S) +$$
$$+ 0.585 \cos(334^{\circ}.060-4M_M+2M_S) + 0.204 \cos(100^{\circ}.800-5M_M+3M_S) +$$
$$+ 0.154 \cos(227^{\circ}.400-6M_M+4M_S) + 0.101 \cos(96^{\circ}.300-6M_M+3M_S) +$$
$$+ 0.106 \cos(222^{\circ}.700-7M_M+4M_S) + 0.163 \cos(198^{\circ}.600-M_J+2M_S) +$$
$$+ 7.208 \cos(179^{\circ}.532-M_J+M_S) + 2.600 \cos(263^{\circ}.217-M_J) +$$
$$+ 2.731 \cos(87^{\circ}.145-2M_J+2M_S) + 1.610 \cos(109^{\circ}.493-2M_J+M_S) +$$
$$+ 0.164 \cos(170^{\circ}.500-3M_J+3M_S) + 0.556 \cos(82^{\circ}.650-3M_J+2M_S) +$$
$$+ 0.210 \cos(98^{\circ}.500-3M_J+M_S) + 0.419 \cos(100^{\circ}.580-M_{Sn}+M_S) +$$
$$+ 0.320 \cos(269^{\circ}.460-M_{Sn}) + 0.108 \cos(290^{\circ}.600-2M_{Sn}+2M_S) +$$
$$+ 0.112 \cos(293^{\circ}.600-2M_{Sn}+M_S) \text{ (arcsec.)}$$

$\Delta\lambda_N$ = nutation in longitude (see § 1.4)

2.2 True Ecliptic Latitude of the Sun

β = true (geometric) latitude of the sun

$= \Delta\beta_{mn} + \Delta\beta_p$

where

$\Delta\beta_{mn}$ = lunar perturbations in latitude

$= 0.576 \sin F$ (arcsec.)

$\Delta\beta_p$ = planetary perturbations in latitude

$= 0.210 \cos(151^\circ\!.8+3M_V-4M_S) + 0.166 \cos(265^\circ\!.5-2M_J+M_S)$ (arcsec.)

2.3 True Radius Vector of the Sun

R = true (geometric) radius vector of sun

$= 10^{\log R}$,

where

logR = natural logarithm of radius vector

$= \log\bar{R} + d(\log R)_{mn} + d(\log R)_p$

$\log\bar{R}$ = logarithm of mean radius vector

$= 0.00003057 + (-0.00727412 + 0.00001814\ T_e) \cos M_S -$

$- 0.00009138 \cos 2M_S - 0.00000145 \cos 3M_S$

$\Delta(\log R)_{mn}$ = lunar perturbations in logarithm of radius vector

$= 0.00000134 \cos D$

$\Delta(\log R)_p$ = planetary perturbations in logarithm of radius vector

$= 0.00000236 \cos(209^\circ\!.080+M_V-M_S) + 0.00000684 \cos(58^\circ\!.318+2M_V-M_S) +$

$+ 0.00000087 \cos(226^\circ\!.700+2M_V-3M_S) + 0.00000105 \cos(87^\circ\!.570+3M_V-3M_S) +$

$+ 0.00000150 \cos(255^\circ\!.250+3M_V-4M_S) + 0.00000206 \cos(253^\circ\!.828-2M_M+2M_S) +$

$+ 0.00000707 \cos(89^\circ\!.545-M_J+M_S)$

2.4 Annual Aberration Correction to the Ecliptic Longitude

λ_A= apparent longitude of sun corrected for annual aberration

$= \lambda + \Delta\lambda_A$

β_A= apparent latitude of sun corrected for annual aberration

$= \beta$,

where

$\Delta\lambda_A$ = annual aberration correction to longitude

$= 20.496 \ (e \cos (\omega - \lambda_s) - 1)$

2.5 Conversion from Ecliptic to Right Ascension System

α_A = apparent right ascension of sun corrected for annual aberration

$$= \arctan\left(\frac{\cos\beta_A \sin\lambda_A \cos\varepsilon - \sin\beta_A \sin\varepsilon}{\cos\beta_A \cos\lambda_A}\right)$$

δ_A = apparent declination of sun corrected for annual aberration

$$= \arcsin\left(\cos\beta_A \sin\lambda_A \sin\varepsilon + \sin\beta_A \cos\varepsilon\right) .$$

2.6 Diurnal Aberration Correction

α_D = apparent right ascension of sun corrected for diurnal aberration

$= \alpha_A + \Delta\alpha_D$

α_D = apparent declination of sun corrected for diurnal aberration

$= \delta_A + \Delta\delta_D$,

where

$\Delta\alpha_D$ = diurnal aberration correction to right ascension

$= 0.32 \cos\Phi \cosh_A \sec\delta_A$ (arcsec.)

$\Delta\delta_A$ = diurnal aberration correction to declination

$= 0.32 \cos\Phi \sinh_A \sin\delta_A$ (arcsec.)

2.6 Diurnal Aberration Correction (cont'd)

h_A = apparent hour angle of sun corrected for annual aberration

= GAST - α_A .

Φ = observer's geographic latitude.

2.7 Semi-Diameter Correction to the Horizontal Angle

HA = horizontal angle to centre of sun

= $HA^{obs} + \Delta HA$,

where

HA^{obs} = observed horizontal angle measured clockwise from the reference object to the trailing edge of the sun

ΔHA = semi-diameter correction to observed horizontal angle

= $0.266994 / (R \cos a_D)$ (deg.)

R = true (geometric) radius vector of sun (see § 2.3)

a_D = apparent altitude of sun corrected for diurnal aberration

= $\arcsin(\sin\delta_D \sin\Phi + \cos\delta_D \cos h_D \cos\Phi)$

h_D = apparent hour angle of sun corrected for diurnal aberration

= GAST - α_D .

3. Polaris Update

The expressions given in this section can be found in most textbooks on geodetic astronomy. Polaris is updated from the 1975.0 catalogued mean position given by the Fourth Fundamental Catalogue (FK4) prepared by Fricke and Kopff [1963].

3.1 FK4 Catalogued 1975.0 Data for Polaris

α_c = catalogued mean right ascension

= 2.12403~~78~~50 (hr.)

μ_α = catalogued centenial proper motion in right ascension

= 0.0056894 (hr./century)

$d\mu_\alpha/dt$ = catalogued centenial variation in proper motion in right ascension

= 0.0028261 (hr/century^2)

δ_c = catalogued mean declination

= 89.1499556 (deg.)

μ_δ = catalogued centenial proper motion in declination

= -0.0002167 (deg./century)

$d\mu_\alpha/dt$ = catalogued centenial variation in proper motion in declination

= -0.0004389 (deg./century^2).

3.2 Time Interval

t = interval of tropical centuries elapsed since 1975.0

= (JD - 2442413.478) /36524.2199 (tropical centuries)

d

3.3 Removal of the E-terms of Aberration from the Catalogued Coordinates

α_o = right ascension of polaris for 1975.0 uncorrected for annual elliptical aberration

$= \alpha_c - \Delta(\alpha_o)_E$ = 31.858302 (deg) = 2.1238868 (hrs)

Removal of the E-terms of Aberration from catalogued coordinates (cont'd)

δ_o = declination of polaris for 1975.0 uncorrected for annual elliptical aberration

$= \delta_c - \Delta(\delta_o)_E$, = 89.149867 (deg)

where

$\Delta(\alpha_o)_E$ = annual elliptical aberration correction to right ascension for 1975.0

-0.343

$= ~~-20.496~~ \sec\delta_c \, (\cos\omega_o' \cos\varepsilon \cos\alpha_c + \sin\omega_o' \sin\alpha_c)$ (arcsec)

$\Delta(\delta_o)_E$ = annual elliptical aberration correction to declination for 1975-0

-0.343

$= ~~-20.496~~ [\cos\omega_o' \cos\varepsilon \, (\tan\varepsilon \cos\delta_c - \sin\delta_c \sin\alpha_c) +$

$+ \cos\alpha_c \sin\delta_c \sin\omega_o']$ (arcsec.)

ω_o' = ecliptic longitude of solar perigee for 1975.0

= 28~~3~~2.512750 (deg.) .

3.4 Proper Motion Correction

α_{PM} = right ascension of polaris corrected for proper motion

$= \alpha_o + \mu_\alpha t + 0.5 \, (d\mu_\alpha/dt) t^2$

δ_{PM} = declination of polaris corrected for proper motion

$= \delta_o + \mu_\delta t + 0.5 \, (d\mu_\delta/dt) t^2$.

3.5 Precession Correction

α_P = Right ascension of polaris corrected for precession

$$= \arctan\left(\frac{\cos\delta_{PM}\ \sin(\alpha_{PM} + \zeta)}{\cos\theta\ \cos\delta_{PM}\ \cos(\alpha_{PM}+\zeta) - \sin\theta\ \sin\delta_{PM}}\right) + z$$

δ_P = declination of polaris corrected for precession

$$= \text{arc sin}\ (\sin\theta\ \cos\delta_{PM}\ \cos\ (\alpha_{PM}+\zeta) + \cos\theta\ \sin\delta_{PM}) ,$$

where

$$\zeta = 2305.297\ t + 0.302\ t^2\ \text{(arcsec.)}$$

$$z = \zeta + 0.791\ t^2\ \text{(arcsec)}$$

$$\theta = 2004.042\ t - 0.426\ t^2\ \text{(arcsec.)} .$$

3.6 Nutation Correction

α_N = right ascension of polaris corrected for nutation

$$= \arctan\left(\frac{N(2,1)\ \cos\delta_P\ \cos\alpha_P + N(2,2)\ \cos\delta_P\ \sin\alpha_P + N(2,3)\ \sin\delta_P}{N(1,1)\ \cos\delta_P\ \cos\alpha_P + N(1,2)\ \cos\delta_P\ \sin\alpha_P + N(1,3)\ \sin\delta_P}\right)$$

δ_N = declination of polaris corrected for nutation

$$= \arcsin\left(N(3,1)\ \cos\delta_P\ \cos\alpha_P + N(3,2)\ \cos\delta_P\ \sin\alpha_P + N(3,3)\ \sin\delta_P\right)$$

where

$$N(1,1) = \cos(\Delta\lambda_N)$$

$$N(1,2) = -\sin(\Delta\lambda_N)\ \cos\bar{\varepsilon}$$

$$N(1,3) = -\sin(\Delta\lambda_N)\ \sin\bar{\varepsilon}$$

$$N(2,1) = \cos(\bar{\varepsilon}+\Delta\varepsilon_N)\ \sin(\Delta\lambda_N)$$

$$N(2,2) = \cos(\bar{\varepsilon}+\Delta\varepsilon_N)\ \cos(\Delta\lambda_N)\ \cos\bar{\varepsilon} + \sin(\bar{\varepsilon}+\Delta\varepsilon_N)\ \sin\bar{\varepsilon}$$

$$N(2,3) = \cos(\bar{\varepsilon}+\Delta\varepsilon_N)\ \cos(\Delta\lambda_N)\ \sin\bar{\varepsilon} - \sin(\bar{\varepsilon}+\Delta\varepsilon_N)\ \cos\bar{\varepsilon}$$

3.6 Nutation Correction (cont'd)

$N(3,1) = \sin(\overline{\varepsilon}+\Delta\varepsilon_N) \quad \sin(\Delta\lambda_N)$

$N(3,2) = \sin(\overline{\varepsilon}+\Delta\varepsilon_N) \quad \cos(\Delta\lambda_N) \cos\overline{\varepsilon} - \cos(\overline{\varepsilon}+\Delta\varepsilon_N) \sin\overline{\varepsilon}$

$N(3,3) = \sin(\overline{\varepsilon}+\Delta\varepsilon_N) \quad \cos(\Delta\lambda_N) \sin\overline{\varepsilon} + \cos(\overline{\varepsilon}+\Delta\varepsilon_N) \cos\overline{\varepsilon}$

3.7 Annual Circular Aberration Correction

α_A = apparent right ascension corrected for annual circular aberration

$= \alpha_N + \Delta\alpha_A$

δ_A = apparent declination corrected for annual circular aberration

$= \delta_N + \Delta\delta_A$,

where

$\Delta\alpha_A$ = annual circular aberration correction to right ascension

$= -20.496 \sec\delta_N(\cos\lambda_s \cos\varepsilon \cos\alpha_N + \sin\lambda_s \sin\alpha_N)$ (arcsec.)

$\Delta\delta_A$ = annual circular aberration correction to declination

$= -20.496 \left[\cos\lambda_s \cos\varepsilon(\tan\delta_N - \sin\delta_N \sin\alpha_N) + \cos\alpha_N \sin\delta_N \sin\lambda_s\right]$ (arcsec.)

λ_s = ecliptic longitude of sun

$\doteq L + C$ (See § 2.1) .

$\varepsilon = \overline{\varepsilon} + \Delta\varepsilon$

3.8 Annual Elliptical Aberration Correction

α_E = apparent right ascension corrected for annual elliptical aberration

$= \alpha_A + \Delta\alpha_E$

δ_E = apparent declination corrected for annual elliptical aberration

$= \delta_A + \Delta\delta_E$,

3.8 Annual Eliptical Aberration Correction (cont'd)

where $\Delta\alpha_E$ = annual elliptical aberration correction to right ascension

$= \sout{-20.496}\ -0.343\ \sec\delta_A\ (\cos\omega'_A\ \cos\varepsilon\ \cos\alpha_A + \sin\omega'_A\ \sin\alpha_A)$ (arcsec.)

$\Delta\delta_E$ = annual elliptical aberration correction to declination

$= \sout{-20.496}\ -0.343\ [\cos\omega'_A\ \cos\varepsilon\ (\tan\varepsilon_A\ \cos\delta_A - \sin\delta_A\ \sin\alpha_A) + \cos\alpha_A\ \sin\delta_A\ \sin\omega']$

(arcsec.)

ω'_A = ecliptic longitude of solar perigee for α_A and δ_A (see § 1.3).

3.9 Diurnal Aberration Correction and Final Apparent Coordinates

α = final apparent right ascension corrected for diurnal aberration

$= \alpha_E + \Delta\alpha_D$

δ = final apparent declination corrected for diurnal aberration

$= \delta_E + \Delta\delta_D$,

where

$\Delta\alpha_D$ = diurnal aberration correction to right ascension

$= 0.32\ \cos\Phi\ \cos h_E\ \sec\delta_E$

$\Delta\delta_D$ = diurnal aberration correction to declination

$= 0.32\ \cos\Phi\ \sin h_E\ \sin\delta_E$

h_E = apparent hour angle corrected for annual aberration

$= GAST - \alpha_E$,

4. Azimuth Determination

4.1 Hour Angle Solution

The azimuth of the sun or polaris, and thus of the reference object, may be derived from the hour angle system by transforming the updated (apparent) astronomic coordinates of the sun or polaris into the horizon system, i.e.,

$$Az = \text{azimuth of sun or polaris}$$

$$= \arctan\left(\frac{-\cos\delta\ \sin h}{\sin\delta\ \cos\Phi - \cos\delta\ \cos h\ \sin\Phi}\right) = \arctan\left(\frac{n}{d}\right) ,$$

$$a = \text{apparent altitude}$$

$$= \arcsin(\sin\delta + \sin\Phi + \cos\delta\ \cos h\ \cos\Phi) ,$$

where

$$h = \text{apparent local hour angle}$$

$$= GAST - \alpha - \lambda_w$$

$$\Phi = \text{observer's astronomic latitude}$$

$$\lambda_w = \text{observer's astronomic west longitude.}$$

For low order work, the observer's approximate geodetic position is substituted for the astronomic position.

The sign of the azimuth is determined by the signs of the numerator(n) and denominator(d) of the arctan argument. The "rule" for the correct sign is;

(i) if $n < 0$,

$$Az = \arctan\left(\frac{n}{d}\right) + 180 \text{ (deg)}$$

(ii) if $n \geq 0$ and $d < 0$,

$$Az = \arctan\left(\frac{n}{d}\right) + 360 \text{ (deg)}$$

(iii) otherwise,

$$Az = \arctan\left(\frac{n}{d}\right) .$$

This problem is avoided if either a rectangular-to-polar conversion or a FORTRAN ATAN2 function is available.

4.2 Zenith Distance Solution

If the altitude or zenith distance of the sun or polaris is observed instead of precise time, the following equations can be used to compute the astronomic azimuth. An approximate time (UT) must be used to derive the apparent astronomic coordinates of the sun or polaris.

Az = apparent azimuth of sun or polaris

$$= \arccos\left(\frac{\sin\delta - \sin\Phi\ \sin a}{\cos\Phi\ \cos a}\right),$$

where

Φ = observer's astronomic latitude

a = observed altitude of sun or polaris (corrected for geocentric parallax for observations on the sun , i.e., $a = a^{obs} + \Delta a_{par}$)

Δa_{par} = geocentric parallax of solar altitude

$\doteq + \arccos(\sin\Pi\ \cos a^{obs})$ $\doteq \Pi \cos a^{obs}$ (arcsec)

Π = constant of parallax

= 8.794 (arcsec.) •

4.3 Azimuth of the Reference Object

Az_{RO} = apparent azimuth of the sun

$= Az_{sun} - HA$

where,

Az_{sun} = apparent azimuth of the sun

HA = observed horizontal angle measured clockwise from the reference object to the sun. If the trailing edge of the sun is observed the semi-diameter correction must be applied to HA (see §2.6) .

5. Concluding Remarks

The expressions provided in this paper will result in an accuracy of better than 1 second of time in right ascension and 1 second of arc in declination for both the sun and polaris. Greater accuracies are possible if more terms in the expressions for $\Delta\lambda_P$ and $\Delta\lambda_N$ are included. The extra terms may be obtained from Craymer [1984] or Newcomb [1898].

The same expressions for polaris may also be applied to other stars where their catalogued FK4 data are used in place of those for polaris (see § 3.1).

6. References

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